

Quiz 5

January 29, 2016

1. The rate of change of a population is jointly proportional to the current population and the square root of the population limit minus the current population. The population limit is 2,000. Write a differential equation, using k for a constant, P for the population, and t for time, which describes this situation.
2. Solve the initial value problem: $y' = xe^{x^2}y$ and $y(0) = 3$.

$$1. \frac{dP}{dt} = kP(2000 - P)^{1/2}$$

$$2. \frac{dy}{dx} = xe^{x^2}y$$

$$\int \frac{1}{y} dy = \int xe^{x^2} dx$$

$$\ln|y| = \int xe^{x^2} dx$$

$$\ln|y| = \frac{1}{2}e^{x^2} + C$$

$$|y| = e^C e^{\frac{1}{2}e^{x^2}}$$

$$y = Ae^{\frac{1}{2}e^{x^2}}$$

$$\text{@ } x=0: 3 = Ae^{\frac{1}{2}e^{0^2}}$$

$$3 = Ae^{\frac{1}{2}}$$

$$A = \frac{3}{e^{1/2}}$$

$$y = \frac{3}{e^{1/2}} e^{\frac{1}{2}e^{x^2}} = \boxed{3e^{\frac{1}{2}e^{x^2} - \frac{1}{2}}}$$